Non-Hermitean Random Matrices: 50 Years After Ginibre
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Abstracts

22-01

Quantum graphs and random matrix theory

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For completely connected simple graphs with incommensurate bond lengths and with unitary or orthogonal symmetry we prove the Bohigas-Giannoni-Schmit conjecture in its most general form. For graphs that are classically mixing (i.e., for which the spectrum of the classical Perron-Frobenius operator possesses a finite gap), we show that the generating functions for all (P,Q) correlation functions for both closed and open graphs coincide (in the limit of infinite graph size) with the corresponding expressions of random matrix theory .

22-02

Random matrix theory for resonances in chaotic scattering: A short overview

Yan Fyodorov ⌖ Queen Mary University of London ✉ y.fyodorov@qmul.ac.uk

I am going to discuss the main results about statistics of S-matrix poles (resonances) obtained in the last 26 years in the framework of effective random matrix non-Hermitian Hamiltonian approach to quantum chaotic scattering.

22-03

Elastic enhancement factor as an indicator of peculiarities of the internal dynamics of open quantum systems

Valentin Sokolov ⌖ Budker Institute of Nuclear Physics ✉ valya.sokolov@gmail.com

Excess of probabilities of the elastic processes over the inelastic ones is a common feature of the resonance phenomena described in the framework of the random matrix theory. This phenomenon is quantitatively characterized by the *elastic enhancement factor* that is a typical ratio F of elastic and inelastic cross sections. Being measured experimentally, this quantity can provide information to us on the character of dynamics of the intermediate complicated open system. I'll discuss peculiarities of the enhancement factor in a wide scope from mesoscopic systems to macroscopic analogous devices.

23-01

Majorana zero-modes and the Ginibre ensemble

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The real Ginibre ensemble exhibits a condensation of eigenvalues on the real axis, such that on average $\sqrt{N}$ of the N eigenvalues have zero imaginary part. This phenomenon has a physical manifestation in the appearance of Majorana zero-modes in superconducting quantum dots.

Background reading:

C. W. J. Beenakker, *Random-matrix theory of Majorana fermions and topological superconductors*, arXiv: 1407.2131.

23-02

Non-Hermitian transport and topological protection in photonic systems

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Optical systems can acquire non-Hermitian properties due to leakage as well as internal losses or amplification. This renders the frequencies of modes complex, and also induces non-orthogonality of modes. This talk explores the consequences for the dynamics and transport in quasi-one-dimensional coupled-resonator wave guides. In this setting, I show that the group velocity differs from the transport velocity of the intensity, and describe the formation of topologically protected modes that are immune to losses. I also mention two-dimensional generalizations of these ideas.

Background reading:

H. Schomerus and J. Wiersig, *Non-Hermitian transport effects in coupled-resonator optical waveguides*, arXiv: 1409.5037

H. Schomerus, *Topologically protected midgap states in complex photonic lattices,* arXiv: 1301.0777

C. Poli, M. Bellec, U. Kuhl, F. Mortessagne and H. Schomerus, *Selective enhancement of topologically induced interface states*, arXiv: 1407.3703

C. Poli, M. Bellec, U. Kuhl, F. Mortessagne and H. Schomerus, *Parity anomaly and Landau-level lasing in strained photonic honeycomb lattices*, arXiv:1208.2901

23-03

Statistics of reflection eigenvalues of chaotic cavities with non-ideal leads

Pedro Vidal ⌖ Universität Bielefeld ✉ vidal.pedro@gmail.com

Random Matrix Theory is a powerful tool to describe the electric and thermal transport properties of quantum dots and Andreev quantum dots. One approach is through the statistics of the reflection/transmission eigenvalues of the scattering matrix. In this talk we will consider non-ideal quantum dots of symmetry class A, D and C. We will mainly focus on the derivations of the joint probability density function of reflection eigenvalues using Jack polynomials, Color-Flavor transformations and different representations of hypegeometric functions of matrix argument. For the class A the reflection eigenvalues will characterize electric transport while for the D and C symmetry classes they will describe thermal transport.

23-04

Non-Hermitian matrix approach to disordered tight-binding models

Felix Izrailev ⌖ Universidad Autónoma de Puebla ✉ felix.izrailev@gmail.com

We study scattering properties of 1D and quasi-1D disordered tight-binding models described by non-Hermitian random matrices. In contrast with full random matrices, our models depend on the degree of internal chaos measured in terms of localization length. Another key parameter is the strength of coupling to continuum, or, the same, the degree of openness of the models. Besides that, we show how the loss or gain (or both) can be included in the model thus allowing studying their influence on transport properties. Main attention is paid to the transmission and reflection coefficients in dependence on the key parameters, as well as to the distribution of poles of scattering matrices. A particular interest is in an emergence of the so-called "superradiance" occurring when the resonances are strongly overlapped.

23-05

Euclidean random matrices for waves in random media

Sergey Skipetrov ⌖ Université Grenoble ✉ sergey.skipetrov@lpmmc.cnrs.fr

In the past, standard random matrix ensembles such as the Gaussian and Wishart ensembles, for example, were very helpful to understand global statistical properties of disordered wave systems. However, a rigorous consideration of wave scattering by a collection of scatterers leads to a more exotic class of matrices, the so-called Euclidean random matrices (ERM). I will review our recent work on the use of ERM to study such problems as superradiance in large atomic ensembles, random lasing, and Anderson localization. An emphasis will be put on open physical problems for which ERM may provide a new insight and eventually a solution.

23-06

Dysonian dynamics of the Ginibre ensemble

Maciej Nowak ⌖ Jagiellonian University ✉ nowak@th.if.uj.edu.pl

We study the time evolution of Ginibre matrices whose elements undergo Brownian motion. The non-Hermitian character of the Ginibre ensemble binds the dynamics of eigenvalues to the evolution of eigenvectors in a non-trivial way, leading to a system of coupled nonlinear equations resembling those for turbulent systems. We formulate a mathematical framework allowing simultaneous description of the flow of eigenvalues and eigenvectors, and we unravel a hidden dynamics as a function of new complex variable, which in the standard description is treated as a regulator only. We solve the evolution equations for large matrices and demonstrate that the non-analytic behavior of the Green's functions is associated with a shock wave stemming from a Burgers-like equation describing correlations of eigenvectors. We conjecture that the hidden dynamics, that we observe for the Ginibre ensemble, is a general feature of non-Hermitian random matrix models and is relevant to related physical applications.

23-07

Ginibre evolutions and interacting particle systems

Oleg Zaboronski ⌖ University of Warwick ✉ o.v.zaboronski@warwick.ac.uk

In this talk I will report some results of our study of Brownian motion with values in $N \times N$ real matrices and the induced evolution of real eigenvalues (Ginibre process). In particular, I will discuss the relation between annihilating Brownian motions in one dimensions and the Ginibre process. I will also show how to calculate multi-time correlation functions for the real eigenvalues and present some mathematical structures arising in the computation.

23-08

Ginibre ensembles and the BKP hierarchy

Alexander Orlov ⌖ Shirshov Institute for Oceanology ✉ orlovs55@mail.ru

We consider three Ginibre ensembles (real, complex and real-quaternionic) with a deformed measure and relate them to known integrable systems by presenting partition functions of these ensembles in form of fermionic expectation values. We also introduce double deformed Dyson-Wigner ensembles and compare their fermionic representations with those of Ginibre ensembles.

23-09

Quantum dots and Jack polynomials

Andrzej Jarosz ⌖ Universität zu Köln ✉ jedrekjarosz@gmail.com

The talk will discuss a random-matrix approach to quantum transport in chaotic quantum dots with one non-ideal lead and Dyson's symmetry parameter 1, 2 or 4. The reflection eigenvalues (the fundamental quantities of the theory) are shown to form a novel probability ensemble, described in terms of Jack polynomials, which are objects appearing in various settings in mathematics and physics. This ensemble reveals links to various challenging mathematical questions.

23-10

Universality in spectral statistics of open quantum graphs

Boris Gutkin ⌖ Universität Duisburg-Essen ✉ boris.gutkin@uni-duisburg-essen.de

The spectrum of closed chaotic quantum graphs are known to be universal, and well described by random matrix theory. For open quantum graphs subject to some damping the quantum evolution is determined by non-unitary matrices whose eigenvalues are distributed over the complex plane. I will show that their statistics exhibit universality at the "soft" edges of the spectrum. The same spectral behavior is observed in many classical non-unitary ensembles of random matrices. For large classes of graphs this implies a special form of fractal Weyl law at the edges of the spectrum. The implication of these results for the scattering on quantum graphs will be discussed as well.

24-01

Circular law and its generalizations

Alexander Tikhomirov ⌖ Syktyvkar State University ✉ antikhom51@gmail.com

We state the general results about universality of singular values distribution and eigenvalues distribution of matrix valued functions of independent random matrices and give many examples of applications these results. In particular we consider the convergence of spectral distribution of product of independent rectangular and square random matrices, we find the limit distribution and prove universality under minimal conditions of singular values and eigenvalues distribution of product of independent matrices from spherical ensemble (i.e., matrix of the type XY^{-1} where X and Y are Ginibre–Girko independent matrices). Joint work with F. Goetze and H. Koesters.

24-02

On the concentration of random multilinear forms and the universality of random block matrices

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The circular law asserts that if X_n is a $n \times n$ matrix with iid complex entries of mean zero and unit variance, then the empirical spectral distribution of $X_n / \sqrt{n}$ converges almost surely to the uniform distribution on the unit disk as tends to infinity. Answering a question of Tao, we prove the circular law for a general class of random block matrices with dependent entries. The proof relies on an inverse-type result for the concentration of linear operators and multilinear forms. Joint work with Sean O'Rourke.

24-03

Hole probabilities in the Ginibre ensemble

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The infinite Ginibre ensemble is the determinantal point process in the complex plane defined by the Bargmann-Fock space of entire functions. The hole probability for a set $\mathcal{A}$ is the probability that no points in the point process fall in the set $\mathcal{A}$. We study the hole probability in two settings. (i) $\mathcal{A} = r\mathcal{B}$ where $\mathcal{B}$ is a fixed shape and $r \to \infty$. Here the hole probability is $\exp(-cr^4)$ and goal is to find precise constant c as a function of the shape $\mathcal{B}$. (ii) $\mathcal{A}$ is an arbitrary shape. Here the aim is to find a simpler substitute for the function $-\log(\text{hole probability for } \mathcal{A})$ that is accurate up to constants. Joint work with Kartick Adhikari.

24-04

Numerical range and non-Hermitian random matrices

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For any matrix $\boldsymbol{A}$ of order N one defines its 'numerical range' as a subset of the complex plane, $W(\boldsymbol{A}) = \{z \in C : \langle \psi | \boldsymbol{A} | \psi \rangle = z\}$ for a normalized $|\psi\rangle \in H_N$. For any normal matrix $\boldsymbol{A}$ this set coincides with the convex hull of the spectrum of $\boldsymbol{A}$. A short review of properties of numerical range of small matrices is presented and asymptotic results for random matrices are discussed. We show that for a random Ginibre matrix $\boldsymbol{G}$ with spectrum asymptotically confined in the unit disk, its numerical range forms a disk of radius $\sqrt{2}$. This result is shown to be related to the Dvoretzky theorem and the structure of the set of mixed quantum states of size N.

Background reading:

B. Collins, P. Gawron, A. E. Litvak and K. Życzkowski, *Numerical range for random matrices,* arXiv: 1309.6203

24-05

Matrices of random phases: The unimodular ensemble

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This contribution concerns an ensemble of order N square matrices, $\boldsymbol{A}_N$, whose entries are independent random numbers uniformly distributed on the unit circle. We consider the eigenvalue distribution of $\boldsymbol{\rho}_N = \boldsymbol{A}_N \boldsymbol{A}_N^T / N^2$, and based on analytic calculations of moments $\left\langle \mathrm{Tr}\left(\boldsymbol{\rho}_N^n\right)\right\rangle$ up to order $n = 4$, conjecture a connection to a recently discussed combinatorial construct, the "Borel triangle", which allows for exact evaluation for all n. This also allows for an exact ensemble average of all the Renyi entropies including the von Neumann entropy $-\mathrm{Tr}(\boldsymbol{\rho}_N \log \boldsymbol{\rho}_N)$. The motivations derive from a study of the nonlocal properties of an ensemble of diagonal random unitary matrices, or entangling power of diagonal quantum gates. The study also allows for studying the entanglement of the auxiliary ensemble of phase random states, which in some fixed basis have all coefficients to be random unimodular phases. It is shown how this ensemble compares with the Hilbert-Schmidt ensemble wherein the matrices $\boldsymbol{A}_N$ belong to the Ginibre ensemble, apart from normalization. This talk is based on the collaboration with Z. Puchala and K. Życzkowski .

24-06

A central limit theorem for biorthogonal ensembles

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This talk describes joint work with Maurice Duits, studying asymptotic fluctuations for biorthogonal ensembles with a finite term recurrence. We show that the asymptotics are universal in the sense that they are only determined by right limits of the recurrence matrix, and obtain a central limit theorem for a wide class of biorthogonal ensembles.

26-01

Random Green's operators in disordered open media

Arthur Goetschy ⌖ Institut Langevin - ESPCI ✉ arthur.goetschy@univ-paris-diderot.fr

We present a statistical study of non-Hermitian matrices relevant to wave propagation in disordered open media. Two types of matrices will be considered, that both are built from the Green's function of the wave equation. The first type belongs to the ensemble of Euclidean random matrices. It can be used to study Anderson localization or random lasers made of active point-like scatterers. The second type is appropriate to describe quasi-modes in open systems made of passive scatterers with arbitrary shape. In particular, we will characterize the decay rate statistics of the quasi-modes and show how it can be used to quantitatively estimate the intensity and the number of lasing modes produced by conventional random lasers.

Background reading:

A. Goetschy and S. E. Skipetrov, *Euclidean random matrices and their application in physics*, arXiv:1303.2880

A. Goetschy and S. E. Skipetrov, *Non-Hermitian Euclidean random matrix theory*, arXiv:1102.1850

A. Goetschy and S. E. Skipetrov, *Euclidean matrix theory of random lasing in a cloud of cold atoms*, arXiv:1104.2711

26-02

Cooperative effects and photon localization in atomic gases

Eric Akkermans ⌖ Technion ✉ eric@physics.technion.ac.il

We propose a numerical and analytic study of the interplay between cooperative (superradiance and subradiance) and disorder effects in atomic clouds. In a one-dimensional geometry, where $N >> 1$ atoms are randomly distributed along a line, we present an analytic calculation of the photon escape rates based on the diagonalization of the coupling matrix $U_{ij} = \cos(x_{ij})$, where x_{ij} is the dimensionless random distance between any two atoms. In that case, we show that the single-atom limit is never reached and the photon is always localized within the atomic ensemble. This localization originates from long-range cooperative effects and not from disorder as expected on the basis of the theory of Anderson localization. In two dimensions, we obtain a Marchenko-Pastur law that has been developed for diagonalizing Euclidean random matrices. Finally, we study numerically the spectrum of the non-Hermitian effective Hamiltonian describing the dipolar interaction of the atomic gas with the radiation field. We study both scalar and vectorial radiation fields. We show that for dense gases, the resonance width distribution follows, both in the scalar and vectorial cases, a power law $P(\Gamma) \sim \Gamma^{-4/3}$ that originates from cooperative effects between more than two atoms. This power law is different from the known distribution $P(\Gamma) \sim \Gamma^{-1}$ previously considered as a unambiguous signature of Anderson localization of light in random systems. We also show that in dilute clouds the center of the energy distribution, $P(E)$, is described by a Wigner semicircle law in the scalar and vectorial cases. For dense gases, Wigner semicircle law is replaced in the vectorial case by the Laplace distribution. In all cases, however, $P(E)$ is dominated by cooperative effects.

26-03

Measuring maximal eigenvalue distribution of Wishart random matrices with coupled lasers

Nir Davidson ⌖ Weizmann Institute ✉ nir.davidson@weizmann.ac.il

Our experimental system of large arrays of coupled lasers uses the combination of dissipative coupling and nonlinear gain to “dissipate” itself to a minimal loss state which is the lowest (complex) eigenvalue of a non-Hermitean random matrix. We show this can be used to study Kuramoto dynamics on random graphs, to simulate interesting phases of the XY spin model, revealing e.g. geometric frustration, extreme value statistics of random processes and random matrices, spin glass, crowd synchrony, and clustering, and may be of interest to the community of random matrices.

26-04

A nonlinear analogue of May-Wigner instability transition

Boris Khoruzhenko ⌖ Queen Mary University of London ✉ b.khoruzhenko@qmul.ac.uk

I would like to present a joint work with Yan Fyodorov suggesting a nonlinear analogue of May-Wigner instability transition. We study a system of $N >> 1$ degrees of freedom individually relaxing with a rate μ and coupled via a smooth stationary random Gaussian vector field with both gradient and divergence-free components. We show that generically with increasing the ratio of the coupling strength to the individual relaxation rate the system experiences an abrupt transition from a topologically trivial phase portrait with a single stable equilibrium into topologically non-trivial regime characterized by exponential in N number of equilibria, vast majority of which is expected to be unstable. Our analysis invokes statistical properties of the elliptic ensemble of real asymmetric matrices, and raises interesting questions about real eigenvalues of such matrices.

26-05

Universal distribution of Lyapunov exponents for products of Ginibre matrices

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There has been recent progress on the distribution of singular values and complex eigenvalues of products of complex Ginibre matrices, giving rise to determinantal expressions for a finite number of matrices T of finite size N. Interpreting the product as discrete time evolution one can define Lyapunov exponents based on the logarithm of the singular values in the large-T limit, that govern the dynamics. These take known deterministic values, and we show that they are normally distributed and compute corrections to the mean and variance. Surprisingly the same values and distributions are obtained to leading order in T, if one repeats the construction with the moduli of the complex eigenvalues of the product matrix instead. In both cases the limiting process is given by a permanent. We conjecture that similar results hold for real and quaternionic matrix elements, where we can only access the complex eigenvalues so far.

26-06

A new approach of SUSY for the complex Ginibre ensemble

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One of the crucial advantages but also of the main problems of the supersymmetry method in random matrix theory is the duality between ordinary random matrix ensembles and supermatrix ensembles. The advantage is that integrals over ordinary matrices of a very large dimension N are equal to integrals over small supermatrices such that the large-N analysis becomes simple. The disadvantage is that the relation between the statistical weights distributing the ordinary matrices and the supermatrices is obscured. Only for Gaussian ensembles the sitation is well understood. But what is with quite general weights, especially when studying more realistic ensembles which may not fall in the known universality classes? Quite recently a projection formula was presented for the eigenvalues of real symmetric, Hermitian and Hermitian self-dual matrices as well as for the singular values of random matrices drawn from the three chiral ensembles which makes such an explicit connection between the ordinary and the superspace. In my talk I will concentrate on the eigenvalue statistics of complex chiral ensembles and show that also for this problem such a projection formula exists.

26-07

Random normal matrices and Kahler metrics

Semyon Klevtsov ⌖ Universität zu Köln ✉ sam.klevtsov@gmail.com

We investigate the analogy between the large-N expansion in normal matrix models and the asymptotic expansion of the determinant of the Hilb map, appearing in the study of critical metrics on complex manifolds via projective embeddings. This analogy helps to understand the geometric meaning of the expansion of matrix model free energy and its relation to gravitational effective actions in two dimensions. We compute the leading terms of the free energy expansion in the pure bulk case, and make some observations on the structure of the expansion to all orders. We will also discuss the generalization of this expansion to the case of beta-ensembles on curved Kahler manifolds.

26-08

Fractional quantum Hall effect in a curved space

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The fractional quantum Hall (FQH) effect occurs when two-dimensional electrons in a high magnetic field condense to form an incompressible quantum liquid. This quantum phase is known to support particle-like excitations with fractional charge and anyonic statistics. A powerful theoretical approach to understanding these properties involves constructing an ansatz for the ground state wave function, and probing the response to variations in magnetic flux and background metric. The resulting universal kinetic coefficients for electromagnetic and gravitational response provide a nearly complete characterization of the FQH state. However, typically only the response to homogeneous deformations is considered, which leaves open the question of how these quantum liquids couple to local curvature and magnetic field gradients. In this talk, we discuss FQH states in the lowest Landau level on more general curved spaces and non-uniform magnetic fields. These states prove to have a rich structure which reveals a close connection between electromagnetic and gravitational responses. We see this by studying local transformation properties of correlation functions under geometric deformations and field variations. We also develop a method for computing the exact asymptotic expansion of these ground state correlation functions.

26-09

Large-*N* limit of Bethe Ansatz wavefunction overlaps

Eldad Bettelheim ⌖ Hebrew University ✉ eldadb@phys.huji.ac.il

In order to be able to describe dynamics for exactly solvable models, the ability to obtain matrix elements of physically relevant operators is crucial. Such calculations in the Bethe ansatz approach are difficult in the thermodynamic limit. We present methods for such calculations, which may allow to extend Bethe ansatz techniques to non-equilibrium.

26-10

Large-*N* expansion of β-ensembles

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We consider a model of logarithmic gas (the β-ensemble) on an arbitrary closed contour in the plane and develop a method for the large-N expansion of the free energy. The main technical tool is an analog of the "loop equation" which has the form of a boundary condition for the stress-energy tensor in a conformal field theory .

27-01

Universality of higher order corrections in random normal matrices

Roman Riser ⌖ Katholieke Universiteit Leuven ✉ roman.riser@wis.kuleuven.be

We will consider $n \times n$ random normal matrices with the probability distribution $P_n(\boldsymbol{M}) = Z_n^{-1} \exp[-n \operatorname{Tr} V(\boldsymbol{M})]$, $V(\boldsymbol{M}) = \boldsymbol{M}^* \boldsymbol{M} - f(\boldsymbol{M}) - f(\boldsymbol{M})^*$. It is known that in the large-n limit, the eigenvalues will fill a domain in the complex plane with constant density. We assume that the potential is such that the averaged density of eigenvalues is supported on a simply connected domain with an analytic boundary. Then there exists a connection with conformal mapping, which was discovered by Wiegmann and Zabrodin in 1999. At the boundary it is known that the density of eigenvalues in zoomed coordinates has an universal behavior which looks like

$$\rho_n\left(z_0 + a\eta / \sqrt{n}\right) = \frac{1}{2\pi} \operatorname{erfc}\left(\sqrt{2}\operatorname{Re} a\right) + O\left(n^{-1/2}\right),$$

where η is the vector normal to the boundary at z_0. We show new results for higher corrections in $1/n$. For example for the potential $V(\boldsymbol{M}) = \boldsymbol{M}^* \boldsymbol{M} - t\,\boldsymbol{M}^2 - \left(t\,\boldsymbol{M}^2\right)^*$ we find

$$\rho_n\left(z_0 + a\eta / \sqrt{n}\right) = \frac{1}{2\pi} \operatorname{erfc}\left(\sqrt{2}\operatorname{Re} a\right) + \frac{\kappa}{\sqrt{2\pi^3 n}} \left(\frac{(\operatorname{Re} a)^2 - 1}{3} - (\operatorname{Im} a)^2 \right) e^{-2(\operatorname{Re} a)^2} + O\left(n^{-1}\right)$$

where κ is the curvature of the boundary at z_0. We will claim that such universal laws can be found in general and we show how one can get expressions for higher order corrections in terms of conformal mappings and the Schwarz function for general potential. As our approach uses orthogonal polynomials, we will also find correction terms for them and show how they can be expressed by different methods.

27-02

Rank one perturbations of Hermitean and unitary β-ensembles

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We apply the theory of orthogonal polynomials on the real line and on the unit circle to deal with rank one perturbations of Hermitian β-ensembles and of unitary circular β-ensembles. We present matrix models, compute the exact joint eigenvalue density, and discuss the asymptotics of the eigenvalues when $n \to \infty$. Joint work with Rowan Killip.

27-03

A discrete random walk approach for the singular values of random Bernoulli matrices

Christopher Joyner ⌖ Weizmann Institute ✉ christopher.joyner@weizmann.ac.il

We investigate the singular values of ensembles of various ensembles of Bernoulli random matrices, i.e. matrices with entries chosen randomly from a discrete set of two elements. Our approach is to construct and analyse a discrete random walk process within the ensemble of such matrices, similar in spirit to Dyson's Brownian motion model but with important modifications. From this we show how, in the limit of large matrix size, one can obtain the joint probability density function for the singular values, which coincides with the well-known fixed trace Gaussian ensembles. Our a pproach is very general and can be adapted to investigate the eigenvalues of discrete matrix ensembles as well as ensembles with non-trivial correlations between the matrix elements.

27-04

High degree singularities in log-gases partition functions

Francesco Mezzadri ⌖ University of Bristol ✉ f.mezzadri@bristol.ac.uk

Partition functions of log-gases whose Boltzmann weight contains an essential singularity appear in many areas of mathematics and physics, like quantum transport, finite-temperature field theory and number theory. Because of such singularities, the asymptotic analysis of these partition functions as the number of particles goes to infinity is a very challenging problem. In recent years there has been progress when the log-gases come from unitary invariant matrix models (Mezzadri and Mo, 2009; Chen and Its, 2010; Brightmore, Mezzadri and Mo, 2014; Xu, Dai and Zhao, 2014). The main feature that seems to arise is that in certain ranges of the perturbation parameter the asymptotics is described by the Painlevé III equation. We will talk about recent progress on the problem.

27-05

Non-Hermitian random matrix theories and QCD

Jac Verbaarschot ⌖ SUNY at Stony Brook ✉ jacobus.verbaarschot@stonybrook.edu

Non-Hermitean random matrix theories have added greatly to two areas of QCD: QCD at nonzero chemical potential and the spectral properties of the Wilson Dirac operator. At nonzero quark chemical potential the QCD Dirac operator does not have any hermiticity properties and its eigenvalues are scattered in the complex plane. Most notably the determinant of the Dirac operator is complex which invalidates stochastic algorithms to evaluate the the path integral. This is an important reason why random matrix methods have had such strong impact on this field. Among others we have understood the quenched approximation, the nature of the sign problem and the relation between chiral symmetry breaking and the spectrum of the Dirac operator. The Wilson Dirac operator is pseudo-Hermitian so that its determinant is real and the theory can be studied by means of Monte Carlo simulations for an even number of flavors. Random matrix theory makes it possible to determine the behavior of the smallest eigenvalues of the Dirac operator. In particular, we have explained puzzling lattice QCD results for the distribution of the topological pseudo zero modes at nonzero lattice spacing. Since the Wilson Dirac operator is pseudo-Hermitian, it can have exactly real eigenvalues. We have determined the distribution of these eigenvalues and have related it to the low energy constants of the effective Lagrangian that describes the discretization effects of the Wilson Dirac operator.

Posters

P-23-01

Noiseless KPZ equation and the complex matrix models

Jacek Grela ⌖ Jagiellonian University ✉ jacekgrela@gmail.com

We deal with a diffusive setup for $\beta = 2$ non-Hermitian matrix models for which an exact finite-N diffusive equation is derived [arXiv: 1405.5244]. By Cole-Hopf transformation it is connected with the noiseless KPZ equation which, in the large-N limit, gives an equation for the mean electrostatic potential – the main object used in obtaining results in the complex matrix models. Utilizing the well-known quaternionic/hermitization method, we obtain formulas for the spectral density, eigenvector correlator and the spectrum boundary for a general initial matrix. These formulas have interesting interpretations of an electrostatic $1/x^2$ system and show a determinantal structure. We conclude by giving some model examples.

Background reading:

J.-P. Blaizot, J. Grela, M. A. Nowak and P. Warchoł, *Diffusion in the space of complex Hermitian matrices - microscopic properties of the averaged characteristic polynomial and the averaged inverse characteristic polynomial*, arXiv: 1405.5244

P-23-02

Time-delay matrix, midgap spectral peak, and thermopower of an Andreev billiard

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We derive the statistics of the time-delay matrix (energy derivative of the scattering matrix) in an ensemble of superconducting quantum dots with chaotic scattering (Andreev billiards), coupled ballistically to M conducting modes (electron-hole modes in a normal metal or Majorana edge modes in a superconductor). As a first application we calculate the density of states ρ_0 at the Fermi level. The ensemble average $\langle \rho_0 \rangle = \delta_0^{-1} M[\max(0, M + 2\alpha/\beta)]^{-1}$ deviates from the bulk value δ_0^{-1} by an amount depending on the Altland-Zirnbauer symmetry indices α, β. The divergent average for $M = 1,2$ in symmetry class $\mathrm{D}(\alpha = -1, \beta = 1)$ originates from the mid-gap spectral peak of a closed quantum dot, but now no longer depends on the presence or absence of a Majorana zero-mode. As a second application we calculate the probability distribution of the thermopower, contrasting the difference for paired and unpaired Majorana edge modes.

P-23-03

Finite size effects and synchronization in randomly connected networks

Luis Carlos Garcia del Molino ⌖ Université Paris Diderot ✉ garciadelmolino@gmail.com

We study how the global emerging behavior of agents interacting randomly is influenced by statistical properties of the connectivity matrix. We consider a model of neuronal network coupled through a random matrix. It is known that increasing the disorder parameter induces a phase transition leading to nontrivial dynamics. Here we present two new results. First, for finite size systems we observe and investigate a novel phenomenon in the sub-critical regime: the probability of observing complex dynamics is maximal for an intermediate system size when the disorder is close enough to criticality. We give a more general explanation of this type of system size resonance in the framework of extreme values theory for eigenvalues of random matrices. Secondly, excitatory-inhibitory structures with balance constraints have been studied in linear systems by deriving the spectral density of the connectivity matrix. We show that in nonlinear systems with such connectivity there is a universal

transition from trivial to synchronized stationary or periodic states lead by the extremal eigenvalues of the connectivity matrix.

Background reading:

H. Sompolinsky, A. Crisanti, and H. Sommers, *Chaos in random neural networks*, Phys. Rev. Lett. **61**, 259 (1988)

G. Wainrib, L. C. Garcia del Molino, *Optimal system size for complex dynamics in random neural networks near criticality*, Chaos **23**, 043134 (2013)

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P-26-01

Fractional quantum Hall effect in a curved space

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I will discuss a general method to compute correlation functions of fractional quantum Hall (FQH) states on a curved space. In a curved space, local transformation properties of FQH states are examined through local geometric variations, which are essentially governed by the gravitational anomaly. Furthermore, we show that the electromagnetic response of FQH states is related to the gravitational response (a response to curvature). Thus, the gravitational anomaly is also seen in the structure factor and the Hall conductance in flat space. The method is based on iteration of a Ward identity obtained for FQH states.

Background reading:

T. Can, M. Laskin, and P. Wiegmann, *Fractional quantum Hall effect in a curved space: Gravitational anomaly and electromagnetic response*, arXiv: 1402.1531

P-26-02

Local properties of products of non-Hermitian random matrices

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We consider products of independent square random non-Hermitian matrices. More precisely, let $\boldsymbol{X}_1,\ldots,\boldsymbol{X}_m$ be independent $N\times N$ random matrices with iid entries with zero mean and variance $1/N$. Soshnikov and O'Rourke showed that the limit of the empirical spectral distribution of the product $\boldsymbol{X}_1\cdot\ldots\cdot\boldsymbol{X}_m$ is supported in the unit disk. We prove that if the entries of the matrices $\boldsymbol{X}_1,\ldots,\boldsymbol{X}_m$ satisfy uniform sub-exponential decay condition, then the spectral radius of $\boldsymbol{X}_1\cdot\ldots\cdot\boldsymbol{X}_m$ converges to unity almost surely as $N\to\infty$.

P-26-03

Diffusing matrices, associated Burgers-like equations and spectral shock waves

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We show that a logarithmic derivative (or a Cole-Hopf transform) of the averaged characteristic polynomial and the averaged inverse characteristic polynomial, associated with Hermitian, Wishart and chiral matrices performing a random walk, satisfy Burgers-like partial differential equations for arbitrary matrix size N and different initial conditions. In all cases, $1/N$ plays the role of viscosity. In the inviscid limit these equations are solved with the method of complex characteristics and the edges of the spectra are associated with shocks. The finite N equations allow to study the microscopic behavior. One recovers Airy, Pearcey and so-called Bessoid functions.

Background reading:

J.-P. Blaizot, J. Grela, M. A. Nowak and P. Warchoł, *Diffusion in the space of complex Hermitian matrices - microscopic properties of the averaged characteristic polynomial and the averaged inverse characteristic polynomial,* arXiv: 1405.5244

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